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Compounded Markups in Complex Market Structures

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Using a publicly available input-output database that covers 44 countries and 56 industries, I show that most prices are, on average, two to three times higher than the natural costs of production, costs that include a normal rate of return to capital. The novelty in this research is the argument that the true markups are compounded—they incorporate the markups already existing in the intermediate goods and services (inputs) that a company purchases in a vertical chain of production. A complex market structure, one in which companies sell and purchase intermediate products from each other in both horizontal and vertical directions, is the perfect environment for inflating a price well above its natural level. This research may help understanding the true extent of market power. Market power has a substantial impact in such matters as income inequality, standard of living, and economic development.

Keywords: *complex market structure, compounded markup, monopoly pricing, world input-output tables*

Introduction

Monopoly pricing is more pervasive than many people think. Virtually every item that is available for purchase incorporates monopoly pricing to some degree through the prices of its intermediate inputs. While consumers do occasionally point to isolated items as being too pricey or being produced by conspicuous monopolies, most of the time consumers just take prices as given (literally) by the goddess of competition and free markets. Consumers' faith in the ability of markets to converge towards the lowest and fairest prices comes from popular theories saying that markets are in general under nobody's interference, efficient, transparent, and highly competitive. The data show otherwise.

In 1776, the founder of modern economics, Adam Smith, explains in memorable words what a natural, fair price would be in a well-functioning economy (Smith 2007, p. 73). He starts by setting the stage: in any society or "neighborhood," there must be "naturally regulated" rates of rent, profit, and wage. Then, Adam Smith continues, "natural" prices of commodities are those prices that just pay for the rent, profits, and labor used to manufacture and bring the products to the market. I call Adam Smith's "natural" price of an intermediate good or service, one that is to be used in the production of other intermediate of final goods, a *pure* cost of production. Unlike the natural price, the market price, Adams Smith explains, normally gravitates towards the one determined by supply and demand, though "exclusive privileges of corporations" may keep the prices

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above the natural rate for a long time ("for ages together," in Adam Smith's language).

Adam Smith's insight into the works of the free market appears, though, to have been lost when Alfred Marshall inflicted scientific rigor upon it, stripping it of much of its real-life flavor (Marshall 1893). Perfect competition (demand and supply) models originating in Marshall's work are very elegant and easy to grasp, but of little relevance in reality because most markets are not even remotely as assumed by such models. The success of these models is most likely owed to their formal perfection and apparent simplicity. Using world input-output data, I show that *compounded*, or true price markups are substantially higher than the conventional ones in all industries, being magnified by the flow of intermediate products down the vertical chain of production, each new transaction in intermediate goods adding an extra layer of markup.

The paper is organized as follows: the next section demonstrates compounded markups in two stylized, very simple examples. The Methodology section gives the theoretical framework for determining conventional markups when sector-level data are available and establishes a formula for compounded markups in input-output data. Data section describes the world input-output datasets used in this paper. Results section shows the calculations and discusses the results. Last section concludes and suggests possible generalizations of complex market structure models. Appendix A gives a simple theoretical framework to show that compounded markups must be greater than the conventional ones. Appendix B extends the idea of markup compounding to a purely vertical market structure.

Two Simple, Hypothetical Examples of Markup Compounding

A Two-Sector, Vertical Market Example

The simplest example of a *complex market structure* would only involve one final sector and one input (an intermediate product). Suppose the natural, economic cost of producing the input is \$5 but it sells for \$6, such that its markup is $6/5 = 1.2$. Suppose the final product uses the \$6 input, plus some \$4 value added, so that its *conventional* cost of production is $6 + 4 = \$10$, and it sells for \$11, so its conventional markup is 1.1. The *compounded* markup of the final good, though, is equal to the final price divided by the sum of all the natural, or pure costs incurred in all stages of the vertical production chain: $\mu'_f = 11/(5 + 4) = 1.2$, which is greater than its conventional counterpart, $\mu_f = 1.1$. The same result can be obtained when pure costs are not known, but the conventional markups are known; all we need to do to retrieve the pure costs is to divide the selling prices by the conventional markups of the intermediate and the final products, as shown in the next hypothetical example. Appendix A gives a more general proof that the compounded markup of the final product must be greater than the conventional one, $\mu'_f > \mu_f$.

The Simplest Input-Output Economy (A Hypothetical Example)

A second, more involved, example simulates an input-output table and explains the calculations of the compounded markup. As I have mentioned, a *complex market structure* involves an intricate network of transactions in intermediate goods and services leading to the production of a final product (see also Colonescu 2021). To better understand the method of markup compounding, let us consider the hypothetical input-output matrix presented in Table 1. I denote the two industrial sectors by *S1* and *S2*; *VA* is value added, *mu* is the conventional markup of each sector, and *MU* is the calculated compounded markup.

A value in Table 1, say the 0.4 in the first row, indicates that sector *S1* sells intermediate inputs to sector *S2* that will make for a share of 0.4 in the total cost of sector *S2*'s output. Thus, the entries in the table (except the *mu* and *MU* columns) are shares of intermediate inputs moving from the row-heading sectors into the column-heading sectors. Companies may purchase inputs from their own sector, inputs that will be subject to the same-sector markup.

Table 1. *An Example of an Input-Output Table with Value Added and Markups*

Sector	S1	S2	mu	MU
S1	0.3	0.4	1.9	2.5
S2	0.5	0.1	1.2	1.5
VA	0.2	0.5	1.0	1.0

The sum of the shares, including the value added, must equal 1 in each column. This observation helps us determine the value added, when it is not known, as the difference between 1 and the sum of the intermediate input shares. I assume the markup (price over marginal cost) for value added is equal to 1—no markup. (The value added comes, for instance, from the contributions of labor and capital, priced at their natural rates.)

Each input in the table consists of two unobservable parts: a *pure*, or *natural* price, and a (conventional) markup. Once the markup is determined in separate calculations, the natural part can be determined as the share shown in the table over the respective sector's markup. For instance, sector 1's *compounded* markup is calculated as the market value of the output, which is equal to $\mu_{S1} \times 1$, over the sum of all the *natural* prices of the inputs. Equation (1) gives the calculation for sector 1. The resulting value of 2.45 for the compounded markup exceeds sector 1's conventional markup of 1.9.

$$\mu_1' = \frac{1.9 \times 1}{(.3/1.9 + .5/1.2 + .2/1.0)} = 2.45 \tag{1}$$

The purpose of this paper is to show that $\mu' > \mu$ is a general result. The observation that the compounded markup is greater than the conventional one may seem trivial, but, to my knowledge, it was never subjected to empirical investigation, nor quantified; moreover, with the rare exception of the double marginalization theory in industrial economics, this market feature is never mentioned.

Method

The first part of the method is not new, but it is necessary for my purpose, so I briefly explain it here. The method serves at calculating industry-level, or *conventional* markups, where the marginal cost consists of the sum of the prices a company pays to purchase the intermediate products it needs, plus the rent it pays to use the machinery, plus wages. The key part at this point is that the intermediate products already contain a markup charged by upstream sectors; the upstream markups are "laundered" in the downstream sectors and incorporated in legitimate costs; in other words, the costs in the current sector already contain markups originating in upstream sectors. Conventional markup calculations disregard these hidden markups.

The second part of my method adds up all the conventional markups incorporated in a final-use commodity. Adding up, however, is a misnomer, because vertical supply chains do not add up, but multiply, or compound, successive markups (see also Appendix B).

A Theory of Conventional Markups

The Cost-Minimization Problem

Following an established literature, such as Hall et al. (1986) and De Loecker and Warzynski (2012), I measure the degree of monopoly pricing by markup, defined as the ratio of price over marginal cost and use the *production approach* developed by the same authors. This method assumes that firms minimize costs in the short run by choosing the amounts of some variable inputs, in particular intermediate products they purchase from upstream sectors. Under this approach, capital is considered fixed, and the production function is homogeneous of degree one (constant returns to scale).

Let us denote the intermediate product used by sector i by I_i, capital by K_i, and the target amount of output by $\bar{F}_i$; the price of the intermediate product purchased by sector i is P_i^I. The price of the intermediate product is a price index, and the quantity of the intermediate input is measured by its dollar value. This way, we can aggregate all inputs from all sectors in one variable in firm i's cost function. The rental rate of capital in sector i is P_i^K. Given the production function $F_i(I_i, K_i)$, the total cost to be minimized is (2) and the Lagrangean function corresponding to the conditional cost minimization problem is (3).

$$TC_i(I_i, K_i) = P_i^I I_i + P_i^K K_i \tag{2}$$
$$\mathcal{L}(I_i, K_i, \lambda_i) = P_i^I I_i + P_i^K K_i - \lambda_i[F_i(I_i, K_i) - \bar{F}_i] \tag{3}$$

With capital being maintained fixed, the only first-order condition of the minimization problem is (4).

$$P_i^I = \lambda_i \frac{\partial F_i(I_i, K_i)}{\partial I_i} \tag{4}$$

In the cost-minimization problem, the Lagrangean multiplier, λ_i, can be interpreted as the change in the total cost at its optimum level when the target output increases by an extra unit (the envelope theorem). In other words, λ_i is the marginal cost of production. The next few equations try to find a simpler form for the markup, μ_i, by manipulating the first order condition (4). With P_i^O denoting the price of output of sector i, equation (4) is equivalent to the following sequence:

$$\frac{P_i^I}{\lambda_i} = \frac{\partial F_i(I_i, K_i)}{\partial I_i} \frac{I_i}{\bar{F}_i} \frac{\bar{F}_i}{I_i} \frac{P_i^O}{P_i^O} \tag{5}$$

$$\frac{P_i^O}{\lambda_i} = \frac{P_i^O \bar{F}_i}{P_i^I I_i} \times \frac{\partial F(I_i, K_i)}{\partial I_i} \frac{I_i}{\bar{F}_i} \tag{6}$$

Let us now introduce the following notations: Call *markup* the ratio of the price of output over marginal cost, $\mu_i = P_i^O / \lambda_i$; denote the share of input expenditure in the value of output by α_i, given by the formula $\alpha_i = P_i^I I_i / (P_i^O \bar{F}_i)$; finally, denote the elasticity of output with respect to input by θ_i, calculated as in the last part of equation (6). With these notations, we can finally write the markup equation for sector i, as in Hall (1988) and followers, as shown in (7). Relationship (7) gives a compact formula to calculate what I call the *conventional* markup.

$$\mu_i = \frac{\theta_i}{\alpha_i} \tag{7}$$

<u>Estimating Output Elasticity</u>

The challenge with the markup described in (7) is to determine θ_i, the elasticity of output with respect to input, which can be done by estimating a constant-elasticity production function. The task is, though, complicated by the endogeneity of the input term in the production function: as Rovigatti (2017a) mentions, a productivity shock affects the dependent variable (output), which, in turn, affects the independent variable (input). Successive efforts by Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg et al. (2006), Ackerberg et al. (2015), and Wooldridge (2009) have refined our knowledge in estimating production functions. The underlying idea, put foreword among the first by Olley and Pakes (1996) and subsequently refined by others, is to use a *control* function.

I use here the Olley and Pakes (1996) approach, as described in Rovigatti (2017a). The production function to be estimated has the form in (8), keeping the notations introduced in (2); Q stands for output and A stands for total factor productivity, $A_j = \exp(\alpha_j + \omega_{ktj})$; j denotes sector, k denotes country, and t denotes year. Each sector works according to the production function (8). The production function in logarithmic form is (9), where u_{ktj} is an identically and independently distributed error term.

$$Q_{ktj} = A_j I_{ktj}^{\theta_j} K_{ktj}^{\gamma_j} \tag{8}$$

$$q_{ktj} = \alpha_j + \theta_j i_{ktj} + \gamma_j k_{ktj} + \omega_{ktj} + u_{ktj} \tag{9}$$

To estimate (9), Olley and Pakes (1996) propose modeling the state variables in the problem, such as capital and the idiosyncratic error term ω, as a polynomial function, $\Phi(k_{ktj}, g_{ktj})$), where ω has been replaced by a proxy, a known variable, g_{ktj}. With this, the production function to be estimated becomes (10).

$$q_{ktj} = \alpha_j + \theta_j i_{ktj} + \Phi\left(k_{ktj}, g_{ktj}\right) + u_{ktj} \tag{10}$$

The two stages in estimating (10) involve estimating first the control function, Φ, then the parameter of interest, θ_j.

Calculating Compounded (Grand) Markups

Following Miller and Blair (2009), let us use the following notations: Z is the input-output matrix, where the element z_{ij} is the dollar value of industry (sector) i's output used in production by industry j. X is the vector of total output, one element for each industry, measured in dollars. If we divide all elements z_{ij} by their corresponding total output, we obtain the technical coefficients, $a_{ij} = z_{ij}/x_j$. The technical coefficient a_{ij} represents the value of input coming from industry i into industry j necessary to produce \$1 worth of output j. (This \$1 worth represents the cost-value, not the market value of the final product; it is equal to the sum of all costs of production.)

The value of all inputs used in \$1 worth of sector j's output is the sum of all input shares a_{ij} coming from all sectors i into sector j, as in equation (11), where C_j represents the total cost of the intermediate products used in production by sector j, and N is the number of all sectors. Each input a_{ij} includes a markup charged by industry i.

$$C_j = \Sigma_{i=1}^{N} a_{ij} \tag{11}$$

Consumers would like to know how much of this cost is total markup charged by all the sectors providing inputs to sector j. Let μ_i stand for the markup charged by sector i for delivering an intermediate product to sector j; I call this the *conventional* markup because it corresponds to the usual definition of markup. Let us denote by c_i some measure of pure marginal cost in sector i's output, which is the marginal cost striped of any markups charged by upstream industries. Equation (12) gives the relationship among the technical coefficients, pure marginal cost, and the markup ratio.

$$a_{ij} = c_{ij}\mu_i; \quad c_{ij} = \frac{a_{ij}}{\mu_i} \tag{12}$$

The total cost of production of the final good j must include both the cost of inputs (11) and the value added by sector j, and must be equal to 1.

The pure marginal cost of the output of sector j is equal to the sum of all pure marginal costs of all inputs; the grand markup is denoted by μ'_j. Equation (13) is the key relationship that allows calculating μ'_j, the *grand* markup of sector j. Each side in (13) represents the market value of sector j's output.

$$\mu'_j \times (\text{pure cost})_j = \mu_j \times (\text{conventional cost})_j \tag{13}$$

The conventional cost in (13) is equal to 1, such that the formula for the grand markup is (14).

$$\mu'_j = \frac{\mu_j}{(\text{pure cost})_j} \tag{14}$$

If the value added in the final sector is included in its total cost as just another input, then the compounded markup can be calculated using the remarkably simple equation (15), which is a generalization of equation (1).

$$\mu'_j = \frac{\mu_j}{\sum_{i=1}^{N} \frac{a_{ij}}{\mu_i}} \tag{15}$$

The goal of this research is to calculate the grand markups, μ'_j, for all country-sector entries in the world input-output database (WIOD 2018). While the technical coefficients, a_{ij}, are easily available through simple operations on the input-output tables, calculating the conventional markup ratios, μ_j, given by (16), requires estimating the regression equation (10). Equation (16) is the same as (7) but written for sector j.

$$\mu_j = \frac{\theta_j}{\alpha_j} \tag{16}$$

Data

I use the socio-economic accounts part of the world input-output database (WIOD-SEA 2016) for calculating the *conventional* markups, μ_j, and the 2009 full input-output table (WIOD 2018) for calculating the compounded markups, μ'_j. The database covers 44 countries and 56 industries (sectors). Thus, the input-output table is a square matrix, Z, of dimensions $2\,464 \times 2\,464$, a matrix with over six million data entries. Each row in the matrix is a country-sector item and has a corresponding column. The z_{ij} entry in the input-output table is the value of an intermediate product sold by the country-sector i and purchased by the country-sector j to be used in the final product of the country-sector j. The notations in this part follow Miller and Blair (2009) and the authors of the R package *ioanalysis*, Wade and Sarmiento-Barbieri (2020).

As I have mentioned, each entry in the input-output table represents the dollar value of the respective input. If this value is divided by the value of the final product, the result is the share of the input in output, which is denoted by a_{ij}; matrix A is the matrix of all elements a_{ij}, also called *technical coefficients* in the input-output jargon.

Table 2 provides descriptive statistics for a very small part of matrix A, the matrix of technical coefficients. All the elements of this matrix must be less than 1, with many being zero or close to zero, because an industry in a country will only purchase inputs from a relatively small number of the other country-industry entities. The column names in Table 2 stand for *Austria*, followed by a number indicating the industry, according to the *RNr* column in Table 3.

Table 2. *Descriptive Statistics for the Technical Coefficients (Sample)*

	AUS 1	AUS 2	AUS 3	AUS 4	AUS 5
Number of values	2 464.0	2 464.0	2 464.0	2 464.0	2 464.0
Number of zeros	221.00	272.00	234.000	230.00	235.00
Max	0.183 9	0.089 0	0.072 0	0.043 7	0.301 1
Range	0.183 9	0.089 0	0.072 0	0.043 7	0.301 1
Sum	0.555 4	0.357 8	0.379 4	0.421 1	0.699 2
Mean	0.000 2	0.000 1	0.000 2	0.000 2	0.000 3
Std. deviation	0.004 0	0.002 8	0.002 0	0.001 8	0.006 7

Table 3. *Industry Descriptions and Codes*

ISIC industry code	Industry description	RNr
AtB	Agriculture, hunting, forestry, and fishing	1
C	Mining and quarrying	2
15t16	Food, beverages, and tobacco	3
17t18	Textiles and textile products	4
19	Leather and footwear	5

Results

To calculate the conventional markups, I use, first, the function *prodestOP* in the *R* software package *prodest* (Rovigatti 2017b), which estimates the regression equation (10) following the two-stage method by Olley and Pakes (1996). Equation (17) shows the practical version of the theoretical equation (10). The symbols in (17) are chosen to correspond to those in the database; they represent the following variables: *GO_QI* represents output, *II_QI* represents intermediate inputs (the adjustable input variable), K_GFCF represents capital (the state variable), and the ratio *GF_CF/GFCF_P* represents investment (the control variable in the two-stage method). Equation (17) is estimated in logarithms. Table 4 shows some descriptive statistics of the variables used in (17), as they appear in the database. The values of each variable in the socio-economic part of the database and represented in Table 4 (17) are given for each country-sector observation.

$$GO\_QI_{ktj} = \alpha_j + \theta_j \; II\_QI_{ktj} + \Phi(K\_GFCF_{ktj}\, , GF\_CF_{ktj}/GFCF\_P_{ktj}) + u_{ktj} \tag{17}$$

Table 4. *WIOD-SEA: Descriptive Statistics of Selected Variables*

Variable	Description	Nr. val.	Median	Mean
GO_QI	Gross output, volume indices	21 360	119	145
II_QI	Intermediate inputs, volume indices	21 015	122	158
K_GFCF	Real fixed capital stock, mil.	20 045	15 912	6 125 139
GFCF	Gross fixed capital formation, mil.	20 415	1 802	538 521
GFCF_P	Price level of *GFCF*	20 008	110	313

Now that we have the estimated values of θ for each sector, we use (16) to calculate the conventional markups at country-sector level, and, with these, the *compounded* markups (15). For one country-sector observation, the sum in the denominator of (15) extends over all inputs coming from all other countries and sectors; value added is included in this sum as just another input. Table 5 shows a sample of the compounded markups by sectors, averaged over all countries.

Table 5. *Average Compounded and Conventional Markups by Sector (Sample)*

Sector description	Compounded markup	Conventional markup
Real estate activities	3.16	2.31
Financial intermediation	3.19	2.15
Education	4.22	2.08
Renting of m&eq and other business activities	1.28	1.94
Inland transport	2.13	1.63
Mining and quarrying	4.38	1.51
Health and social work	2.08	1.43
Air transport	5.42	1.41
Rubber and plastics	3.58	1.37
Water transport	4.60	1.36

The values of the compounded markups in Table 5 are high but plausible, considering they accumulate all markups in all intermediate inputs. In rare instances, the calculations yield conventional markups greater than the compounded ones, as one of the entries in Table 5 appears. This anomaly may be generated by measurement errors and missing data; some of the missing data have been imputed, which also introduces measurement errors. A small number of observations giving obviously erroneous results, such as negative markups or extremely high values have been eliminated. I estimate that such cases make for less than ten percent of all observations.

Even understanding the nature of the compounded markups, many of us will be, probably, surprised by their magnitudes. A large compounded markup may not, however, reflect a high conventional markup in the concerned final-good sector, but it may indicate high-markup inputs in the vertical supply chain leading to the respective final-good sector. Table 6 shows a sample of grand (compounded) markups, averaged by country over all sectors. These are somehow smaller than the sector averages because averages blur large, inter-sector differences.

Table 6. *Average Compounded and Conventional Markups by Country (Sample)*

Country code	Country name	Compounded markup	Conventional markup
IND	India	3.05	2.01
MEX	Mexico	3.63	1.71
GRC	Greece	3.63	1.71
TWN	Taiwan, Province of China	2.27	1.56
BRA	Brazil	2.13	1.49
LTU	Lithuania	3.40	1.47
EST	Estonia	3.21	1.21
BGR	Bulgaria	2.92	1.20
IRL	Ireland	2.86	1.20
KOR	Korea, Republic of	1.74	1.19
HUN	Hungary	2.75	1.19
CZE	Czechia	2.30	1.15

Table 7 shows a comparison between the distributions of the conventional and compounded markups. This table is the key finding of this paper; it shows that the compounded markups are higher than the conventional ones at all quartiles of the distribution. While this result is hardly surprising, the interest is also in the magnitudes of the difference. At the higher end, 25% of the observations show conventional markups greater than 1.52, but compounded markups greater than 3.87; the average conventional markup is 1.44, and the average compounded markup is 3.04.

Table 7. *Comparing the Distributions of Conventional and Compounded Markups*

Markup type	Min	Q25%	Median	Mean	Q75%	Max
Conventional	1	1.13	1.27	1.44	1.52	6.6
Compounded	1	1.60	2.34	3.04	3.87	9.8

Table 8 shows the result of testing the difference in means between the conventional and the compounded markups. The method is a paired t-test, and the hypothesis is H_0: $\bar{\mu} \geq \bar{\mu'}$, H_A: $\bar{\mu} < \bar{\mu'}$, where $\bar{\mu}$ is the sample mean of the conventional markup and $\bar{\mu'}$ is the sample mean of the compounded markup. The test supports the alternative hypothesis that the conventional markup is less than the compounded markup.

Table 8. *Test of Difference in Means between the Conventional and the Compounded Markups*

Estimated difference	Statistic	p-Value	Method	Alternative
-1.3	-24.1	0.0	paired t-test	less

Conclusion

A *grand*, or *compounded* markup, defined as the ratio between price and an unobserved, *pure*, marginal cost, is found to be significantly higher than the conventional markup. While the theory suggests that compounded markups are always greater than the conventional ones, real data may occasionally show exceptions from this rule due to errors or measurement, missing data, and errors of data imputation.

The compounded markups calculated in this study are substantially higher than the conventional ones; and yet they must be underestimated, because only one layer of the vertical production chain is included. In reality, there is a multiplying (compounding) effect in markup when intermediate products travel down the vertical production chain. Thus, the analysis in this research is still incomplete. A realistic theory of a complex market structure should paint a complete picture of the full network of intermediate transactions—a challenging task for future research. Appendix B explores a model of a purely vertical production chain. It shows that, in this case as well, the compounded markup of the final-good sector exceeds the conventional markup.

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Appendix A

Suppose there are N sectors producing intermediate goods, all selling their products to one of them, which is the producer of a final good. In general, companies in the final-good sector may buy from each other the final product to be used as just another input; to keep things simple, though, I will assume in what follows that this is not the case. Let us denote the pure cost in sector i by c_i, the price by p_i, the conventional markup by μ_i, and the compounded markup by μ_i'. Markup is defined in general by the ratio of price over pure cost, such that $p_i = c_i\mu_i$. Without loss of generality, all the calculations can be thought of concerning one unit of the final product.

The main claim of this paper is that the compounded markup in a final product is greater than the conventional markup, where the compounded and conventional markups are defined as follows:

$$\text{Conventional markup: } \mu_f = \frac{p_f}{c_f + \Sigma_{i=1}^{N-1} p_i} = \frac{p_f}{c_f + \Sigma_{i=1}^{N-1} c_i\mu_i}$$
$$\text{Compounded markup: } \mu_f' = \frac{p_f}{c_f + \Sigma_{i=1}^{N-1} c_i}$$

It is clear that the compounded markup is greater than the conventional one, since the (conventional) markups of the intermediate products, μ_i, are all greater than 1.

Appendix B

As a further step towards a full model of complex market structures, consider the simple case of a purely vertical production chain, where sector i has a pure (natural) cost c_i and markup μ_i, and sells an intermediate good to sector 2, which sells its product to sector 3, the producer of the final good. As opposed to pure costs, the conventional costs of production in each of the three sectors, C_i, are the following:

$$C_1 = c_1$$
$$C_2 = c_2 + C_1\mu_1 = c_2 + c_1\mu_1$$
$$C_3 = c_3 + C_2\mu_2 = c_3 + c_2\mu_2 + c_1\mu_1\mu_2$$

With p_3 being the market price of the (final) product of sector 3, the conventional markup in the final sector is

$$\mu_3 = \frac{p_3}{C_3} = \frac{p_3}{c_3 + c_2\mu_2 + c_1\mu_1\mu_2}$$

On the other hand, the compounded markup in the final sector is

$$\mu_3' = \frac{p_3}{c_1 + c_2 + c_3}$$

Since the conventional markups μ_1, μ_2, and μ_3 are all greater than 1, it is clear that the compounded markup of sector 3, μ_3', is greater than the conventional one, μ_3, the former having a smaller denominator.

In general, the conventional cost of production in the final sector is

$$C_f = c_f + c_1\mu_1\mu_2 \dots \mu_{N-1} + c_2\mu_2 \dots \mu_{N-1} + \dots + c_{N-1}\mu_{N-1}$$
$$= c_f + \Sigma_{i=1}^{N-1} c_i \Pi_{j=i}^{N-1} \mu_j$$

With this, the conventional markup, with $N - 1$ intermediate sectors in a purely vertical production chain, is

$$\mu_f = \frac{p_f}{c_f + \Sigma_{i=1}^{N-1} c_i \Pi_{j=i}^{N-1} \mu_j}$$

By comparison, the compounded markup, which is the ratio between the final price and the sum of the pure costs of all upstream stages, is

$$\mu_f' = \frac{p_f}{c_f + \Sigma_{i=1}^{N-1} c_i}$$

Comparing μ_f to μ_f' in the last two equations, it can be observed that μ_f', the compounded markup, is greater than μ_f, the conventional one. The longer the vertical chain of intermediate-good producers, the greater the difference between the true (compounded) markup of the final product and the conventional markup.