JAZZ HARMONY: PITCH-CLASS SET GENERA, TRANSFORMATION, AND PRACTICAL MUSIC

[Reader: please see the examples document that accompanies this paper]

Contemporary jazz musicians, composers and arrangers draw from a rich and varied collection of techniques and strategies used towards the realization of chord progressions and successions, voicing and voice leading, and melody. These techniques include chord and scale substitutions (for function, colour, and quality), and chromatic interactions such as interpolations and substitutions. While jazz musicians recognize the distinctiveness of the various chord-scales and harmonies, they also talk about source scales and chord families, and describe processes that apply to chord and chord-scale substitution.

Building on the transformational system and the model of set-class space introduced in my dissertation, "Transformation and generic interaction in the early serial music of Igor Stravinsky," this paper presents a model of set-class space that is germane to realm of jazz harmony, melody, and linear improvisation. The scales and chords that are typical of the harmonic/melodic language of jazz coalesce into a generic model in which pcset genera form inter-generic relations through transformation.

The system that associates scales and harmonies through this model as set-classes and their concomitant pcsets and subsets engages canonical and non-canonical transformations including TTOs, rotation, interpolation and omission, and near-equivalency. Ultimately, the model suggests a holistic, theoretical definition of jazz harmony and offers musicians a way of thinking about relations among jazz scales and harmonies in terms of a *transformational system* that resides in *transformational space*, which in turn can be employed systematically and imaginatively towards the creation, interpretation and presentation of jazz music.

A *transformational system* simply represents the totality of transformational processes discussed in the present study. The term *system* implies that all of the transformational processes can be grouped together in the non-mathematical sense of a group, and that these processes can interact with one another. Determining which transformation or transformations establish close relationships among musical objects is the same as discovering the transformation or transformations that act upon a musical object that in turn produces images of that object. Thus, modeling relationships among specific pc objects does not need to engage the entire apparatus of a transformational system.

I defined *transformational space* in my dissertation as a conceptual space in which transformations among pc objects take place. (Richards 2003, V, p.199.) Musicians engage in transformational space when they employ transformational processes to any kind of musical object.

Genus, pcset genus, set-class genus, referential collection and source scale have analogous meanings with subtle differences. The concept of a source scale as a referential collection and the concept of a pcset genus (or sc genus) intersect on several salient points. Richard Parks, in his article "Pitch-Class Set Genera," provides the definition of a *simple genus* (that is, a *simple set-class genus*) and points up the close relationship between referential collection and genus. The jazz-theoretic notion of the source scale as used in the present study is analogous to genus, which is congruent with Parks' definitions and desiderata.

While the naming of transformational processes may vary when comparing similar ideas in jazz-theoretic discourse to pitch-class set theoretic discourse, many transformational types are familiar to music students as well as experienced musicians. **[EXAMPLE 1]** illustrates typical instances of interpolation as it applies to the collection of be-bop scales (aka additive scales), which entails the addition of goal-directed chromatic pitches to the ascending and descending iterations of diatonic modes. Other familiar examples include the addition of a chromatic pitch to minor and major forms of the pentatonic scale, resulting in the blues scale and its major version.

Chromatic interactions within diatonic modes suggest how relationships among diatonic modes as well as relationships among other source-scales oriented to the diatonic genus can be established through the non-canonical *near-equivalency* transformation. "Near-equivalency" is a term I introduced in my dissertation, which is a special transformational process that relates to both intersection and mapping. Near-equivalency is derived from Joseph Straus's *near-inversion* and *near-transposition*, which are refinements of Allen Forte's Rp relation. In essence, all of these terms describe near-mappings of pcsets. Near-equivalency connotes the Rp relation in both literal and abstract manifestations. In the context of the present study, near-equivalency represents the transformation of one pcset into another by substituting one pitch-class.

Near-equivalency is an important transformation in terms of establishing *intra-* and *inter*-generic relationships. If near-equivalent transform partners remain inclusion related to the set class that defines a genus, the transformation is *intra-generic*; if the near-equivalent transformation results

in changing the genus-membership of one of the partners, the transformation is *inter-generic*. Near-equivalency can be applied to chords and scales, and is a useful tool for learning and understanding chord and scale structures, and for gauging familial relationships among specific chord-scales and basic seventh-chords in terms of substitution strategies used towards harmonic interpretation and improvisation.

[EXAMPLE 2] demonstrates the five-basic 7th-chords and their close relations (sometimes called "complex 7th-chords") in successive near-equivalent transformations. This model illustrates how chord families are determined through invariant *chord guide-tones*, which are the 3rd and 7th of a chord. CGTs provide the basis for distinctiveness among basic 7th-chord types. The example also indicates set-class names for each 7th-chord. The variety of unique scs shown here indicates the distinctive intervallic characteristics of these chords.

Chord-scales engender a variety of upper functions and/or additions and alterations to the foundation triads and seventh chords, which is a result of the distinct interval content of each scale-type. Chord-scales in themselves can provide pitch-sources for melody, arranging, improvisation, and comping, but are not intended to be "silver bullets" that guarantee successful performance and composition. Rather, a chord-scale provides a harmonic center, or reference, that aligns harmony and line, which in turn allows improvisers to gauge invariance and change with the harmony of the moment and/or the key-center, and engage suitable voice leadings as a means of establishing coherence, direction, and musical interest. The most important aspects of a chord-scale in terms of harmonic center are the root and chord-guide tones.

The multifarious chord-scales used in jazz theory and practice coalesce into a small group of source-scales, each of which defines a unique genus. The diatonic genus, sc 7-35, is central to this group.

Rotation and near-equivalency are both used as a means of learning and exploring chord-scales. **[EXAMPLE 3]** presents the modes of C major as rotations of C Ionian. In this instance, the modes of C major are arranged in a rotational array in which the ordered pcs of the C major scale (Ionian mode), <CDEFGAB> undergo transformation through circular permutation without engaging transposition. Thus, the pitch class content of C major remains invariant for all rotations.

While rotation indicates modal position relative to a specific key, organizing modes parallel to a single root through near-equivalency is extremely useful for chord-symbol realization. **[EXAMPLE 4]** presents the diatonic modes parallel to C using the major-scale ruler, which is a "jazz-slang" approach to the description of the intervals for each scale, and aligns these scales to chord families based on four of the five basic 7th-chords. This process is well understood by jazz musicians. Notice that this arrangement of modes on C aligns to the circle of fifths in terms of key signatures (C Lydian, 1# – C Ionian, no #/b – C Mixo, 1b – C Dorian, 2b – etc.).

The ascending form of the melodic minor scale (aka Jazz minor) is an important source-scale in jazz harmony. The typical description of the structure of the Jazz minor scale begins by comparing it to the parallel major scale as a near-equivalent relationship. Similar to the diatonic modes, the modes of the Jazz minor are generally understood as rotations of the source scale. Moreover, the modes of Jazz minor derive their names from the diatonic modes, which indicates how the near-equivalency transformation plays an important role in jazz theory and practice. **[EXAMPLE 5]** illustrates the relationship of the modes of jazz minor to the diatonic modes. Typical chord symbols associated with each of these scales are included in the example. Note that the nomenclature is not completely standardized, so the example includes alternate names for some of the scales.

The harmonic minor scale and its TnI transform-partner, harmonic major, are frequently used in jazz harmony and evince a distinct genus. The typical use of these scales applies to tonic and dominant-function chords, for example, the harmonic-minor dominant and the harmonic-major dominant scales. A special refinement of the harmonic-minor dominant is effected through the addition of a chromatic pitch into the harmonic tetrachord, resulting in the harmonic-minor dominant add #9 scale (aka "Spanish Phrygian"). **[EXAMPLE 6]** illustrates the near-equivalent transformation of diatonic to harmonic minor/harmonic major, and the transformation of harmonic minor dominant to harmonic minor dominant add #9 through interpolation.

[See EXAMPLE 7]
Scales with highly symmetrical successive interval arrays are also of interest to jazz musicians, especially the octatonic scale (i.e., SC 8-28: SIA <12121212>), which is known to jazz musicians as symmetrical dominant. SC 8-28 only yields three distinct pcsets through Tn/TnI, thus there are only three distinct symmetrical dominant scales. Each of these contains four embedded dominant chords with identical intervallic characteristics since all four chord-scales

remain invariant at Tn/TnI, where N = [0, 3, 6, *and* 9]. These scales allow for a variety of vertical colours that intersect with mixolydian, lydian b7 and altered dominant scales, and suggest many voice-leading pathways as dominant-function chords. The roots of the four embedded dominant chords spell a full-diminished 7th chord. Notice that each unique sym-dom pcset contain two sets of tritone substitution partners at T0 and T6, and T3 and T9. The rotation of the successive interval array of the symmetrical dominant scale by one position yields the successive interval array <21212121>, which jazz musicians recognize as the symmetrical diminished scale and is used primarily as a chord scale for non-dominant function dim.7 chords, for example, the pedal dim.7 chord.

Other chord-scales that express high degrees of symmetry include the symmetrical augmented and whole-tone scales. **[EXAMPLE 8]** demonstrates the transformation of harmonic minor/harmonic major to symmetrical augmented (i.e., sc 7-32 to sc 6-20), and the transformation of jazz minor (as lydian b7) to whole-tone (i.e., sc 7-34 to sc 6-35). In each instance, inter-generic relationships are established through two transformational processes: near equivalency and interpolation/omission (I/O).

[EXAMPLE 9] demonstrates a strategy for improvisation based on the successive near-equivalent transformations of G mixolydian to G lydian b7 to G symmetrical dominant to G altered. Note that the processes of near-equivalency and interpolation/omission are involved in the transformations of lydian b7 to symmetrical dominant, and symmetrical dominant to altered. In practice, improvisers might not produce music that is a step-by-step realization of this model. For example, a performer may choose to create a dramatic change in interval content while holding the CGTs invariant by playing in G mixolydian and then in G altered. This model is also significant because it demonstrates the inter-generic relations among these chord-scales. The chord symbols provided to the right of each staff represent typical harmonies associated with each scale shown in the example.

Functional harmonic progressions typical of jazz standards provide excellent vehicles for improvisation for performers and offer opportunities for interpretation to arrangers (of course, in many instances, arranging is also accomplished in real-time through improvised performance). A study of standard progressions provides improvisers with a model of near-equivalent transformations that aligns chord-scale centers with key signatures. **[EXAMPLE 10]** illustrates the near-equivalent transformations of diatonic modal scales into dominant-function scales in the key of C. The transformations are determined by changing the condition of one of the CGTs

of a diatonic modal scale so that the new scale expresses dominant chord guide-tones (in other words, by creating a leading tone or altering the seventh so that the resultant scale functions as a secondary-dominant). Since each transformation entails a single pitch-class substitution, musicians often describe this process as "inside" because each scale retains a maximum degree of invariance with the tonic key-scale. Notice that this method draws from three referential collections: the diatonic, the jazz-minor, and the harmonic minor/harmonic major.

Jazz compositions and arrangements that are not organized through functional harmony provide challenges and opportunities for performers. Characteristics of non-functional harmonic successions include the lack of a clear tonal center, unusual root progressions and voice-leadings, or an unfamiliar harmonic syntax. The generic approach to realizing such chord successions provides a basis for interpretation upon which the performer draws from the various chord-scales that express the basic chord symbol and create voice leadings in opportunist ways. **[EXAMPLE 11]** illustrates a possible chord-scale realization for an unusual harmonic succession found in "Time Remembered" by Bill Evans. In this case, the relationships among these scales are completely intra-generic.

Instances of passages or entire pieces in which the harmonic rhythm is primarily static also present challenges and opportunities. Static harmonic regions are found in examples of modal jazz music, or introductions, interludes and extended vamp-type endings added to standard pieces or new compositions. **[EXAMPLE 12-A]** illustrates a typical approach to realizing a static harmonic region on C. In this case, the performer selects scales that retain the pitches of Cmi7, but effect change in all other respects by alternating between C dorian and C phrygian, with emphasis on the upper structures. Similarly, a common approach to interpreting the tonic minor is based on intra- and inter-generic potential achieved through near-equivalency processes. **[As shown in EXAMPLE 12-B]**, a static tonic minor chord may be realized as dorian, jazz minor, harmonic minor, natural minor, and so on, depending on the how the player hears the resultant tensions and sets up the voice leading. In this case, the foundation root and minor third are held invariant, while other chord-scale tones are subjected to change.

The near-equivalency transformation plays an important role in chord-substitution techniques. For example, subdominant-function chords often undergo quality and root substitutions. **[In EXAMPLE 13]**, the ii-minor chord in a major tonality changes to minor7b5 though the borrowing of the scale-degree b6 from parallel minor. The root of the transformed chord, in turn, can be substituted with scale-degree b7 through descending third relations while retaining all of the

pitches of ii min7b5 chord so that the ii min7b5 chord becomes the upper structure of the resultant bVII dominant 9 chord.

Transformations such as near-equivalency and interpolation/omission can be applied to linear harmonic structure. Jazz musicians often employ a reductive approach to the harmonic structure of a standard tune in order to introduce static harmonic regions, which allows for more flexibility towards exploring a particular chord-scale or utilizing a variety of chord-scales as a strategy for improvisation. Conversely, the same musicians will interpret regions of static harmony by interpolating harmonies in order to create more harmonic density. Near-equivalency transformations at this level can also a have profound effect on the harmonic quality of a musical passage. For example, interpreting a passage of primarily major-diatonic harmony as minor modality, or imposing quality substitutions for all chords in a passage can yield a fresh or unusual result.

Other transformations typically employed by jazz musicians include the tritone substitution of dominant chords. Jazz musicians understand that a dominant chord can be substituted by another dominant chord that shares the same CGTs, the root of which is at the interval of a tritone from the original chord root. In terms of TTOs, this operation is T6. There are only six tritone partners in the 12-tone universe since two dominant chords share the same tritone formed by the CGTs. The tritone-substitution partners shown in **[EXAMPLE 14]** are notated as root and CGTs and rendered as ordered pcsets, all of which belong to sc 3-8. At T6, the ordered set manifests a unique pc for the first element while elements 2 and 3 switch positions. Thus, the root of the chord transposes by tritone and the CGTs remain invariant, but what was the 3rd becomes the 7th, and vice versa.

The role of the TTO multiplicative function M5 in jazz improvisation is not generally understood by jazz musicians in terms of pcset theory and 12-tone theory, yet players frequently refer to the chromatic genus through melodic figurations that lean on the blues aesthetic, or use chromatic passing tones and other voice-leading devices in goal-directed lines, or use chromatic passages and substitutions as a means of creating tension within diatonic environments. **[EXAMPLE 15]** illustrates a progression comprising ii-V components that are subjected to tritone substitution (for the dominant-function chords), resulting in a chromatic root progression. In other words, tritone substitution transforms cycle-of-fifth progressions to descending chromatic progressions. Tritone substitution chord symbols often include the notation for "add #11," which is the root of the original dominant chord now embedded in the upper structure of the tritone-sub chord. The

boundary interval of sc 7-1 is ic6, the tritone, which aligns with the ic6 relationship inherent to TT-substitution, and indicates the theoretical significance of sc 7-35 transformed to sc 7-1 through M5.

(Conclusion: A Generic Model of Set-Class Space for Jazz Harmony)

[EXAMPLE 16] illustrates the fluid, generic model of set-class space for jazz harmony set forth in the present study. Given the primacy of the diatonic genus, sc 7-35 occupies the central position in this model. The other primary source scales, represented as set classes accompanied by their familiar source-scale names, are aligned to sc 7-35 through the near-equivalency and interpolation/omission transformations. The exception is the M5-transform partnership of scs 7-35 and 7-1. Of course these scales and transformational processes are not exclusive to jazz music. As a collection of scales associated through transformations and aligned to jazz theory and practice, this model does provide a holistic description of jazz harmony in set-class and transformational spaces, and has the potential to include other distinct source-scale types that are also of interest to jazz musicians.

Works cited (also included in the examples document):

Parks, Richard S. "Pitch-Class Set Genera: My Theory, Forte's Theory." *Music Analysis* 17/ii (1998): 206-26.

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Jazz Harmony: Pitch-Class Set Genera, Transformation, and Practical Music
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Example 1: chromatic interpolations applied to diatonic scales

1a: major be-bop (additive), ♯6 ascending, ♭6 descending

(intensifies G as goal)

(intensifies C as goal)

1b: Blues scale (minor pentatonic add ♭5)   1c: major blues scale (major pentatonic add ♭3)

(♭5)   (♭3)

Example 2: 7th chords/chord families in NE relations

	Column 1: The five basic 7th-chords (curved lines indicate NE transform)	Column 2: The five basic and seven "complex" 7th-chords in successive NE relations	Column 3: Chord symbol (and set-class name)
major 7			Ma7 (sc 4-20)
			Ma7 ♭5 (sc 4-15)
			Ma7 ♯5 (sc 4-19)
dominant 7			7 ♯5 (sc 4-24)
			7 ♭5 (sc 4-25)
			7 (sc 4-27)
			7sus4 (sc 4-23)
minor 7			mi7 (sc 4-26)
			miMa7 (sc 4-19)
half-diminished (minor 7 ♭5)			mi7♭5 (sc 4-27)
full-diminished			dimMa7 (sc 4-18)
			dim7 (sc 4-28)

Example 3: diatonic modes and chord-scale functions in C major, sc 7-35

- slurs indicate the arpeggiated foundation 7th-chords
- CGT = chord guide tones, 3rd and 7th

I - C ionian   II - D dorian   III - E phrygian   IV - F lydian   V - G mixolydian   VI - A aeolian   VII - B locrian

typical chord symbols --> associated with each mode

I - C major 7	II - D minor 7	III - E minor 7	IV - F major 7	V - G7 (dominant 7)	VI - A minor 7	VII - B mi7 ♭5 (half-diminished)
CMa7(9,13)	Dmi9(11,13)	Emi7(11)	FMa7(9,♯11,13)	G7(9,13)	Ami9(11)	Bmi7(b5)add11
C6(add9)	Dmi6(add9)			G7(sus4,9)		

Example 4: diatonic modes on C in NE relations, sc 7-35

IV - C lydian   I - C ionian   V - C mixolydian   II - C dorian   VI - C aeolian   III - C phrygian   VII - C locrian

- ♯4, ♭2, ♭3, ♭5, ♭6, ♭7 indicate alterations to the major scale (in "jazz slang")
- slurs indicate NE transformation
- roman numerals indicate modal position

Example 5: diatonic modes and modes of jazz minor in NE relationships, sc 7-35 <-> sc 7-34

I - C ionian   II - D dorian   III - E phrygian   IV - F lydian   V - G mixolydian   VI - A aeolian   VII - B locrian

CmiMa7   Dmi7(♭9,sus4)   E♭Ma7(♯5)   F7(♯11)   G9(♭13)   Ami9(♭5)   B7♯5 (♯9)

(E♭ = D♯)

I - C jazz minor (asc. melodic minor)

II - D dorian ♭2

III - E♭ lydian augmented

IV - F lydian ♭7 (lydian dominant)

V - G melodic dominant (mixolydian ♭6)

VI - A locrian ♮2

VII - B altered (aka: super locrian) (aka: diminished whole-tone)

Example 6: diatonic (sc 7-35) and harmonic minor/major (sc 7-32) in NE realtionship

C aeolian (sc 7-35)   C harmonic minor (sc 7-32)   C ionian (sc 7-35)   C harmonic major (sc 7-32)

G harmonic minor dominant   G harmonic major dominant

G harmonic minor dominant add ♯9 (aka: Spanish phrygian)

(interpolated pc = ♯9)

(William Richards, MacEwan University, Canadian University Music Society/Congress 2012)

Example 7: symmetrical dominant/diminished (octatonic, sc 8-28)

7a. C, E♭, F♯, A sym. dom., 8-28 {0134679t}
- (slurs indicate the chord tones for C7)

C7 (♭9, ♯9, ♯11, 13)   E♭7 (♭9, ♯9, ♯11, 13)   F♯7 (♭9, ♯9, ♯11, 13)   A7 (♭9, ♯9, ♯11, 13)   <-- embedded dominant chords and extensions

SIA: 1   2   1   2   1   2   1   2

extensions ------>   ♭9   ♯9   ♯11   13
(relative to C7)

CGT (3rd and 7th = E and B♭)

7b. C, E♭, G♭, A sym. dim., 8-28 {0235689e}
- (slurs indicate the chord tones for Cdim7; dashed slur indicates alternate 7th for CdimMa7)

Cdim7(9)   E♭dim7(9)   G♭dim7(9)   Adim7(9)
CdimMa7(9)   E♭dimMa7(9)   G♭dimMa7(9)   AdimMa7(9)

SIA: 2   1   2   1   2   1   2   1

(9, relative to Cdim7)

Example 8: transformation of harmonic major/minor to symmetrical augmented (sc 7-32 <-> sc 6-20)
and jazz minor (as lydian ♭7) to whole-tone (sc 7-34 <-> sc 6-35)

8a: sc 7-32 <-> sc 6-20

C harmonic-minor dominant (sc 7-32)

I/O   NE

SIA: 1   3   1   3   1   3

C symmetrical augmented (sc 6-20)

8b: sc 7-34 <-> sc 6-35

C lydian ♭7 (sc 7-34)

I/O   NE

2   2   2   2   2   2

C whole-tone scale (sc 6-35)

note: I/O = interpolation/omision

Example 9: transformation of dominant scales: mixolydian to lydian ♭7 to symmetrical dominant to altered
(sc 7-35 <-> sc 7-34 <-> sc 8-28 <-> sc 7-34)

CGT

G mixolydian
(sc 7-35)   G7, G7sus4, G9, G13, G9sus4

NE

G lydian ♭7
(sc 7-34)   G7(♯11 or ♭5), G9(♯11), G13(♯11)

♯11

NE
I/O

G sym.dom.
(sc 8-28)   G13(♭9), G13(♯9), G13(♭9,♯11)

♭9/♯9

NE
I/O

G altered
(sc 7-34)   G7(♯5,♯9), G7(♯5,♭9)

♭13 (= ♯5)

(William Richards, MacEwan University, Canadian University Music Society/Congress 2012)

Example 10: transformation of diatonic modes into dominant-type scales (in C major)

C ionian (tonic major) — F lydian (IV Ma7#11) — B locrian (vii mi7♭5) — E phrygian (iii mi7) — A aeolian (vi mi7) — D dorian (ii mi7) — G mixolydian (V7)

NE — NE — NE — NE — NE — NE — (invariant)

(E♭ = D♯)

C mixolydian (V/IV scale) — F lydian ♭7 (TTsub V/iii scale) — B altered (V/iii scale) — E har.mi. dominant (V/vi scale) — A melodic dominant (V/ii scale) — D mixolydian (V/V scale) — G mixolydian (V scale)

Example 11: chord-scale strategy for Bill Evan's "Time Remembered"(mm.1-8)

Bmi9 — CMa7(#11) — FMa7 — Emi9 — Ami7 Dmi7 Gmi7 — A♭Ma7(#11) — E♭Ma7(#11)

B dorian / B aeolian — C lydian — F lydian — E dorian / E aeolian — A aeolian/D dorian (D aeolian) / G dorian G aeolian — A♭ lydian — E♭ lydian

Example 12: static harmonic regions and chord-scale strategies

12a: intra-generic transformation (sc 7-35)

C dorian (sc 7-35) — C phrygian (sc 7-35)

Cmi9(11) — D♭Ma7(♭5)/C

E♭Ma7(♭5)/C — F 7sus4/C — D♭Ma7(♭5)/C — D♭Ma7(♭5)/C < -- (2nd inversion of upper structure)

12b: inter-generic transformation (sc 7-35/7-34/7-32/7-35)

C dorian (sc 7-35) — C jazz minor (sc 7-34) — C harmonic minor (sc 7-32) — C aeolian (sc 7-35)

NE — NE — NE — NE — NE

Example 13: subdominant function and NE transformation

Dmi7 — NE — Dmi7(♭5) — B♭7(9)

C major: ii mi7 — ii mi7♭5 — ♭VII7

(function type): subdominant major — subdominant minor — subdominant minor substitution

(William Richards, MacEwan University, Canadian University Music Society/Congress 2012)

Example 14: dominant chords, tritone substitution (T6) and tritone partners

C7	Db7	D7	Eb7	E7	F7

<04t>   <15e>   <260>   <371>   <482>   <593>
T6   T6   T6   T6   T6   T6
<6t4>   <7e5>   <806>   <917>   <t28>   <e39>

Gb7	G7	Ab7	A7	Bb7	B7

Example 15: tritone substitution in progressions (transformation of ic5-cycle to ic1-cycle)

root motion by 5ths (diatonic-based) -->   Emi7   A7   Dmi7   G7
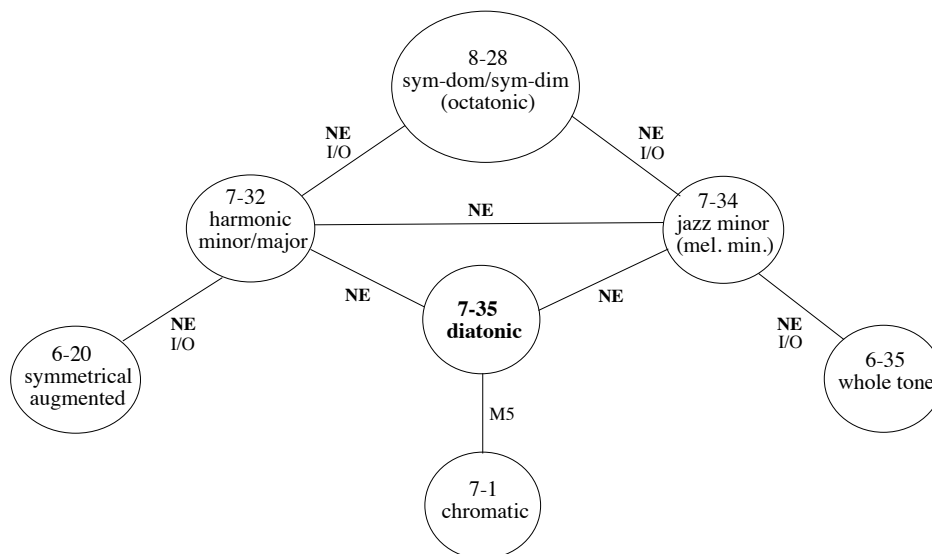
C: ii mi7   V7   ii mi7   V7
                  ii

root motion by ic1 (chromatic) -->   Emi7   Eb7(#11)   Dmi7   Db7(#11)

C: ii mi7   TTsub V7   ii mi7   TTsub V7
                  ii

Example 16: the fluid model of jazz harmony in generic set-class space

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                        8-28
                 sym-dom/sym-dim
                   (octatonic)
          NE                        NE
          I/O                       I/O
     7-32                                 7-34
   harmonic          NE              jazz minor
 minor/major                        (mel. min.)

          NE              NE
   6-20                                        NE
 symmetrical      7-35 diatonic              I/O
 augmented                              6-35
                                      whole tone
                       M5
                      7-1
                    chromatic
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